

INTRODUCING FORWARD SCATTERING IN ADAPTIVE REAR LIGHTING SYSTEMS

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Abstract

Considering recent regulation opportunities to develop adaptive lighting for the automotive industry, this paper shows that a characterization of fog based solely on the atmospheric extinction parameter is not sufficient, especially in the perspective of adaptive lighting for road safety. This has been validated on synthetic images generated with a semi Monte-Carlo ray tracing software dedicated to fog simulation as well as with experiments in a fog chamber. Based on observations showing the limits of classical approaches used to characterize fog, a new way to estimate fog extinction at night with a camera is proposed, along with a method for the classification of fog depending on the forward scattering. Results are given and potential applications are discussed. The main contribution is that it allows for the estimation of a parameter linked to the droplet size distribution of the fog.

Keywords: fog, granulometry, camera, forward scattering, adaptive lighting.

1 Introduction

It is a fact that drivers suffer visibility impairment in fog at night, especially in dense fog when the meteorological visibility distance is no more than a few tens of meters, or when visual cues are scarce. Drivers have been observed to behave inadequately in fog, adopting shorter headway in foggy conditions than they do in clear weather (Caro, 2009).

A first recommendation in order to improve traffic safety in such situations has been to use two rear fog lamps, as far apart as possible, when possible, depending on car design (Cavallo, 2001). It has also been suggested that lowering the mounting height of these lamps could lead to reduced headway estimation (Buchner, 2006).

New proposals are emerging with recent changes in the regulations regarding the intensity of rear lamps (UNECE, 2007). Future adaptive systems will cope with situations more complicated than day/night or tunnel entry/exit differentiation. Technical propositions consist in adapting the intensity and the illuminating area of the lamps. Solutions are being proposed which consist in adapting the intensity of rear lamps to reduced visibility conditions in order to improve perception by maintaining a constant apparent intensity at some distance. In case of fog presence, a common method consists in using the meteorological visibility distance (CIE, 1987), derived from the extinction coefficient k of the atmosphere considered in Beer-Lambert law, in order to compensate for the attenuation of light. In-vehicle prototypes have been developed which use LIDAR technology to estimate k (Luce, 2005).

In this paper, it is proved that knowing k may be insufficient in nighttime fog. Experimentations have been conducted using photometrical simulations (Dumont, 1998) as well as measurements in artificial fog (Colomb, 2008) with radiation and advection fog. These are common types of fog which differ in their droplet size distribution. Significantly different intensities from the same signal with the same meteorological visibility distance depending on the granulometry of the fog were observed: bigger droplets cause stronger forward scattering, and hence higher apparent intensity. This leads us to conclude that using Beer-Lambert model alone may result in inadequate adaptation of lamp intensity and that granulometry needs to be taken into account in adaptive lighting and signalling systems, at least at night.

Consequently, camera- or LIDAR-based estimations of the density of fog at night, which are likely to be used in future ARS (adaptive rear lighting systems), should provide a granulometry related parameter in complement to the meteorological visibility distance which is not sufficient to describe the effects of fog on visual perception. For this purpose, the principle of a camera-based measurement process is proposed which leads to the simultaneous estimation of both fog density and a parameter which quantifies the importance of forward scattering. This process involves three or more light sources at known distances and with known intensities. It has been experimentally assessed and was found to be efficient. However, engineering is still needed to implement this static measurement

process into a real in-vehicle device. Finally, even if the primary application of this work is automotive lighting, the results are of great interest for road lighting, as well as for visual weather monitoring.

2 Model of Light Propagation in Fog

Eq. (1) relates the effects of nighttime fog on photometry from the linear system theory point of view (Nameda, 1992). The first part corresponds to Beer-Lambert's attenuation law for collimated beams. The second part expresses the effect of the scattering of light by the particles in the medium.

$$L_S(d) = L_S(0) \cdot e^{-kd} + L_S(0) * F^{-1} \{ M(k, d) - e^{-kd} \} \quad (1)$$

where $L_S(0)$ is the luminance of the object, k is the extinction coefficient, d is the observation distance and M characterizes the point spread function of fog, F being the Fourier transform. Using the analogy between a slab of fog and an optical filter, the Modulation Transfer Function (MTF) $M(k, d)$ of a homogeneous slab of fog of width d and extinction coefficient k can be derived from the MTF M of a slab of unit optical depth ($kd = 1$), called the frequency contrast operator (FCO) (Dumont, 2004):

$$M(k, d) = M^{kd} \quad (2)$$

In daytime fog a convenient and widely used parameter is the meteorological visibility distance V_{met} . It is related to the extinction coefficient k of Beer-Lambert that is also present in Eq. (1).

V_{met} is defined as :

$$V_{met} = 3/k \quad (3)$$

The use of V_{met} for nighttime fog characterization implies discarding the second part of Eq. (1). This means that the halo effect is neglected. It is shown in section 3.1 that for light sources at night, this model is somehow limited in big-droplet fogs, where forward scattering is significant, because the halo created by forward scattering then has a major impact on the appearance of light sources at night. As mentioned, neglecting the second part of Eq. (1), leads to the Beer-Lambert extinction model:

$$L_S(d) = L_S(0) \cdot e^{-kd} \quad (4)$$

Beer-Lambert describes the first order of interaction between light and the atmosphere. Hence it is a limited model because it does not account for multiple scattering, which is especially a problem in the case of fog, where extinction is mostly due to scattering as absorption is negligible for visible light in water droplets.

3 Fog Simulation and Experimentation

3.1 Fog Simulation by semi-Monte-Carlo Ray Tracing

PROF-LCPC (Photometrical Rendering Of Fog), is a semi-Monte Carlo ray-tracing software developed for the simulation of imaging in fog (Dumont, 1998). Luminance maps of an environment with several light sources in homogeneous fog can be produced. For the interactions of light with fog droplets, tabulated phase functions are used and the extinction factor k of Beer-Lambert model is set from V_{met} using Eq. (3).

In this way, it was possible to calculate the point spread function, and hence the FCO, of two types of fog and then simulate the aspect of traffic in nighttime fogs with the same meteorological visibility distance but with different droplet size distributions, using a generalized version of Eq. (1) (Dumont, 2004). The images presented in Fig. 1 illustrate the impact of fog droplet size on the perception of light sources at night.

Using PROF-LCPC, different configurations were experimented considering the number of light sources and their locations for $V_{met} \leq 500$ m. Variance in the results is proportional to the square root of the number of rays. 10^8 rays were used, which is a common compromise between simulation time and noise.

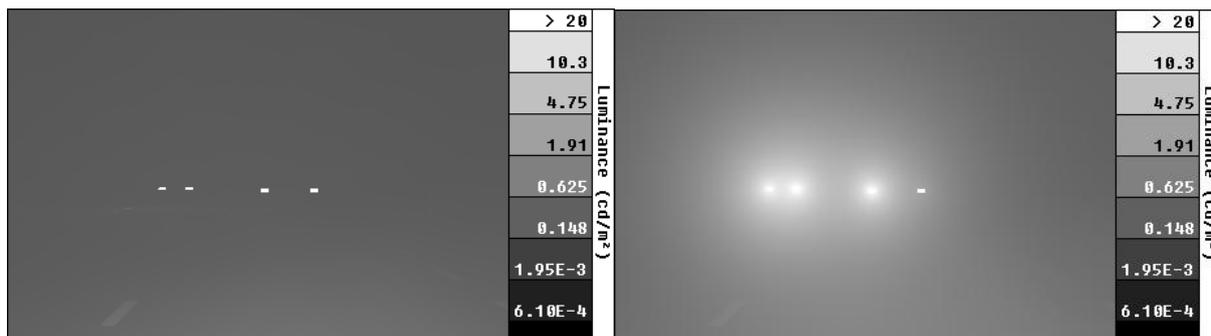


Figure 1. Cars in radiation (left) and advection (right) fog; V_{met} is 100 m in both cases (Dumont, 2004)

3.2 Experimentation in the Fog Chamber

The LRPC of Clermont-Ferrand runs a facility where artificial fog can be produced (Colomb, 2008). It consists in a climatic chamber in which fog is produced by a dedicated system based on water sprinklers. The evolution of the density of the fog is constantly monitored with a TR30 transmissometer from Degreane Horizon (with a 28-m measurement distance). Droplet size distributions are measured with a Palas sensor.

During the experiments, the fog density is first raised to its maximum by saturating the chamber with droplets. Then fog dissipates progressively as heavier droplets fall to the ground and other water droplets aggregate and eventually fall. Because of the nature of the dissipation, fog is stratified, so all the optical instruments and light sources should be placed at the same height.

An experiment was conducted in this fog chamber. Light sources were set at 15 m, 18 m, 23 m and 28 m from the imaging device (see Fig. 2). The light sources were positioned so as to not interact with each other. The experiment consisted in taking pictures with a video-luminancemeter LMK Color 98-4 with a 12 bit CCD sensor while the fog was dissipating. V_{met} values given by the TR30 were simultaneously recorded.

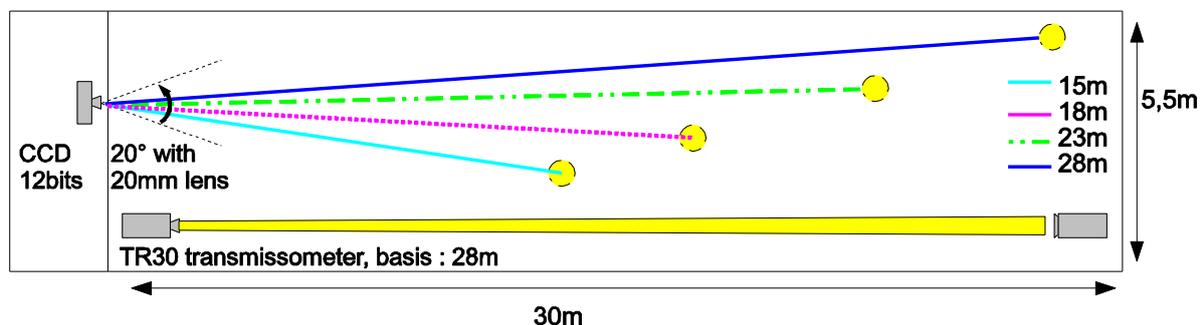


Figure 2. Positioning of the light sources with respect to the camera in the fog chamber

4 Influence of Granulometry on the Intensity of Sources in Fog at Night

Two different sets of phase functions are used hereafter. One set was calculated from Shettle & Fenn droplet size distribution model (Shettle, 1979) illustrated in Fig. 3 (left). Those are denoted G_1 to G_4 (G_1 being the advection fog type and G_4 the radiation fog type). The other was calculated from actual droplet size distributions measured in the fog chamber, presented in Fig. 4 (left). In both cases, the equivalent phase functions were computed using Mie equations.

4.1 Influence of Granulometry in the Simulation Tool

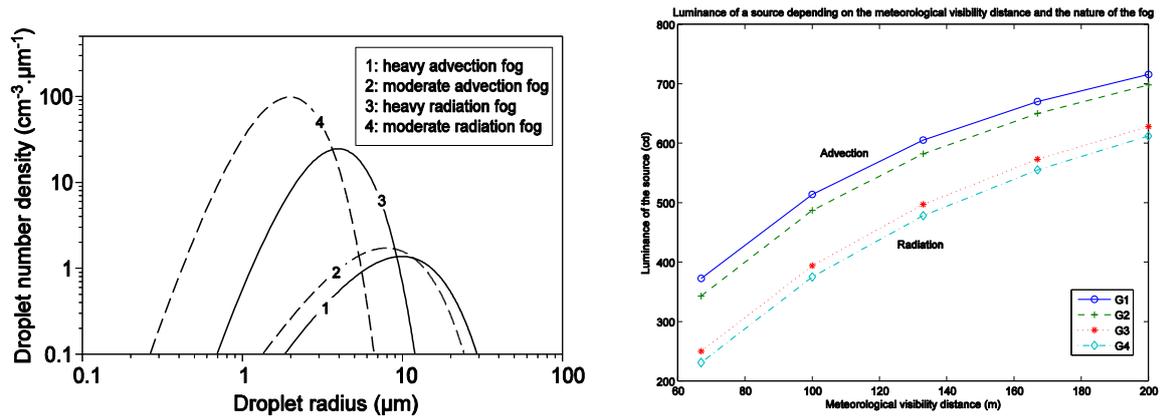


Figure 3. Shettle & Fenn droplet size distribution models (left) and simulated luminance of a lamp at 35 m in the corresponding fogs as a function of V_{met} (right).

The luminance values collected on the simulated luminance maps for a light source at 35 m are shown on Fig. 3 (right) for V_{met} between 66 m and 200 m. It can be observed that luminance in advection fogs is higher than in radiation fogs. This is due to the more important part of oncoming light that is scattered forward by bigger droplets. The relative difference of intensities perceived between radiation and advection fogs may vary from 18 % for $V_{met} = 200$ m to about 75% for $V_{met} = 66$ m. This difference is not negligible, especially when it comes to safety related applications such as adaptive lighting.

4.2 Influence of Granulometry in the Fog Chamber

Different droplet size distributions can be produced in the artificial fog chamber. One fog is produced with tap water, containing minerals, which gives granulometric distributions with a mode around 1 μm and droplets sizes distributed between 0.4 μm and 8 μm, which is characteristic of radiation fog. The other granulometric distribution is obtained with the use of demineralized water, containing less condensation nuclei. This distribution is composed of larger droplets distributed between 0.4 μm and 20 μm and has a mean diameter around 5 μm, which is characteristic of advection fog though it seems natural advection fogs may contain even bigger droplets (Gultepe, 2007; Okuda, 2008).

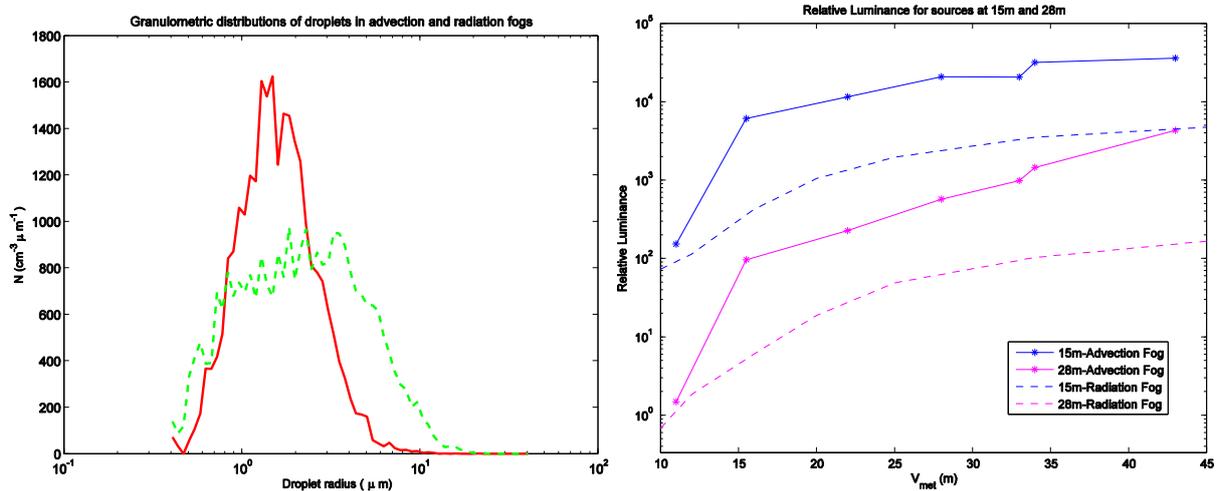


Figure 4. Droplet size distributions measured in the fog chamber (left) and measured luminance of lamps at 15 m and 28 m in the corresponding fogs as a function of V_{met} (right).

Like in the simulation, the relative difference of luminance perceived between radiation and advection fogs is more important for smaller meteorological visibility distances. This relative difference is more important than observed in simulation, but denser fog certainly accounts for this discrepancy. The relative difference varies from 750% for $V_{met} = 45$ m to about 1000% for $V_{met} = 10$ m.

The results obtained from simulation and from actual experimentation show the necessity of taking into account the granulometry of the fog as well as its density.

5 Nighttime Fog Characterization

In the previous section, it was shown that the visual appearance of light sources in fog at night depends on fog density as well as on fog droplet size distribution. It is thus necessary to estimate both parameters to design a relevant ARS. In this section, a preliminary method to estimate both fog density and forward scattering effect is proposed, based on computer vision.

5.1 Estimation of Fog Density using two Light Sources

Using two light sources with luminance $L_i(0)$ and $L_j(0)$ set at different distances d_i and d_j , k can be estimated after simplification if $L_i(0)=L_j(0)$ with Eq. (4):

$$k_{ij} = \frac{\ln\left(\frac{L_i}{L_j}\right)}{(d_j - d_i)} \quad (5)$$

For example, with a pair of light sources located at 80 m and 200 m, different estimations of V_{met} depending on the nature of fog are shown in Fig. 5.

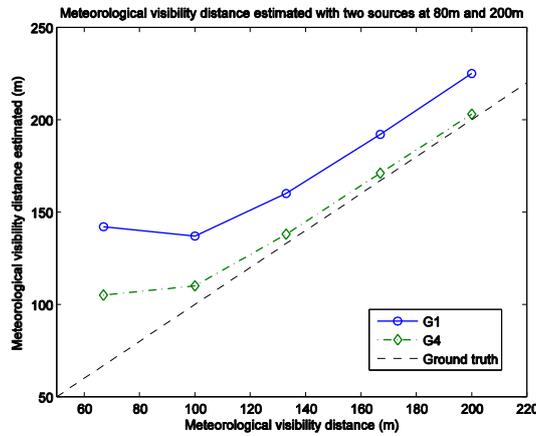


Figure 5. V_{met} estimated from the simulated luminance of a lamp at 35 m in four different fogs as a function of actual V_{met}

For radiation fog like G4 (small particles, mode $\leq 2 \mu\text{m}$), forward scattering is not strong enough to invalidate Beer-Lambert extinction law for $V_{\text{met}} \geq 100$ m. In this example, the error on the estimation of k is less than 10 % with a peak at 50 % for the highest density of fog (the relative error on k equals the relative error on V_{met}).

For advection fog like G₁ (big droplets, mode $\geq 3 \mu\text{m}$), forward scattering is stronger and the error on the estimation of k is higher. It strongly increases for $V_{\text{met}} \leq 100$ m and reaches 100 % for small values of V_{met} . This shows that the estimation of k is biased depending on the position of the light sources and that the bias comes from the scattering of light.

5.2 Estimation of Fog Density using n Sources

Based on the method exposed in Sec. 5.1, the range of fog situations that can be studied is limited by the location of the light sources. This problem may be overcome by placing several sources on a wide range of distance and exploiting the most suitable pair among those available. This method could be used in dynamic conditions provided that the same light source can be observed at different distances. The light sources can be the rear lights of a preceding car or any other lamps in the environment.

Using three light sources, three different estimations of k are computed using the three possible pairs of sources. A method is proposed to automatically extract the most reliable estimation of k based on the notion of sensitivity. Sensitivity is a blind way to estimate the variance of a computation, based on the partial derivatives of a function. Here, we want to know how reliable the estimations are depending on the positioning and the perceived luminance of the light sources. The sensitivity is chosen as the L_2 norm of partial derivatives (Hautiere, 2008):

$$v(k_{ij}) = \left(\frac{\partial k_{ij}}{\partial L_i} \right)^2 + \left(\frac{\partial k_{ij}}{\partial L_j} \right)^2 + \left(\frac{\partial k_{ij}}{\partial d_i} \right)^2 + \left(\frac{\partial k_{ij}}{\partial d_j} \right)^2 \quad (6)$$

k is estimated from the three estimations k_{12} , k_{13} , k_{23} :

$$k = \frac{\sum \frac{k_{ij}}{v_{ij}}}{\sum \frac{1}{v_{ij}}} \quad (7)$$

The sensitivity of V_{met} is estimated with the same principle. Using three light sources at respectively 35 m, 80 m and 200 m, Tab. (1) presents the sensitivities associated to these computations for different values of V_{met} .

Table 1. Sensitivity depending on the pair of light sources observed for different V_{met} in advection fog

V_{met} (m)	v_{12}	v_{23}	v_{13}
33	14	464 173	107 805
100	517	56	459
200	8 441	311	8732

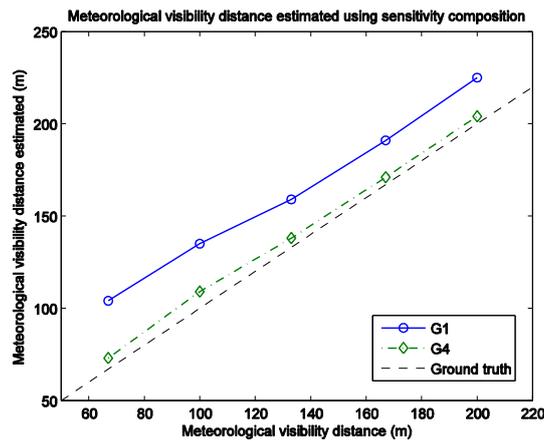


Figure 6. Estimation of V_{met} using three light sources and sensitivity composition

The sensitivity is well suited to the problem. As one can see, the sensitivity is lower for closer light sources (1 and 2) in the heaviest fog ($V_{\text{met}} = 33$ m) and lower for distant light sources (2 and 3) when the fog is lighter ($V_{\text{met}} > 100$ m). In any case, more reliable information can be determined. It works well for radiation fogs (see Fig. 6), but even with sensitivity composition some k values are still inaccurately estimated, particularly in advection fog.

The sensitivity composition of the estimates of k (or V_{met}) can be used with any number of lights at any distances. Supposing several light sources are available, or a given light source can be observed at different distances ranging from 30 m to 400 m or farther, a large range of fog conditions could be addressed.

5.3 The Forward Scattering Bias

5.3.1 Impact of the Forward Scattering

Depending on the size of the droplets, fog may have very different visual effects at night. The presence and size of the halo around light sources depends on the granulometry of fog. The luminance perceived from a light source may differ from Beer-Lambert's extinction law as seen on Fig. 3 (right) and Fig. 4 (right). This leads to biased estimations of the atmospheric extinction parameter and an overestimation of the V_{met} (see part 5.1). In automotive lighting applications, an efficient modulation of lights in foggy conditions relies on an accurate estimation of the meteorological visibility.

In this respect, using an overestimated value of V_{met} , which happens when neglecting forward scattering, may lead to insufficient intensity increase, making the system less efficient.

It is shown in Fig. 6 that even a sensitivity composition does not lead to accurate results in advection fog: 100% error on the estimation of V_{met} in the worst case. The luminance perceived is 60% higher in the big-droplet fog (G_1) than in the small-droplet fog (G_4). This causes V_{met} to be overestimated by 55 %. Using this estimation leads to overestimate the intrinsic luminance $L_i(0)$ of the light sources if we compute it by reversing Eq. (4) following :

$$L_i(0) = L_i(d) \cdot e^{kd_i} \quad (8)$$

Knowing the perceived luminance, the relative error in the estimation of the intrinsic luminance $L_i(0)$ using Eq. (8) was computed. Fig. 7 shows the relative error when computing the luminance of the light sources as a function of V_{met} and distance. This error was tabulated for Shettle & Fenn model fogs (G_1 to G_4 , presented in Fig. 3).

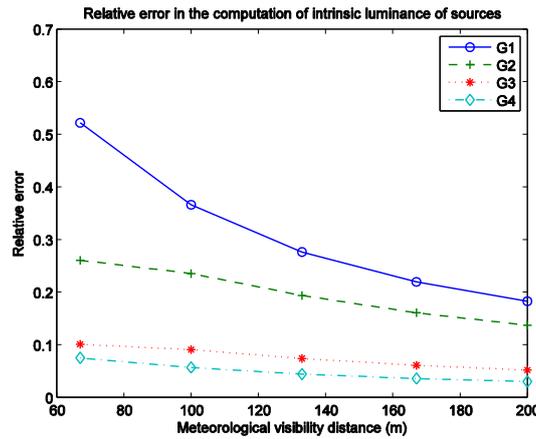


Figure 7. Relative error in the computation of intrinsic luminance with Beer-Lambert law alone

This relative error is independent of the intensity of the light source. Using this error and the estimated V_{met} , the type of fog with respect to its forward scattering properties can be determined.

5.3.2 A Forward Scattering Related Measure: FS

A measure linked to the forward scattering parameter is defined: $FS \in [0;5]$. For an estimated $V_{\text{met-EST}}$, the error is computed and is used to locate it with respect to the four reference error curves (Fig. (7)). Fogs G_4 to G_1 present increasing forward scattering. The measure FS is expected to increase with the error. It is more important for G_1 fog than for G_4 fog. $FS=0$ corresponds to the theoretical case of Beer-Lambert's extinction law. $FS \leq 2$ corresponds to radiation fogs like G_3 or G_4 . $FS \geq 3$ corresponds to advection fogs like G_1 or G_2 . If the relative error on the estimation of intrinsic luminance is more important than that observed for G_1 , FS is clipped to 5. Intermediate values describe the distance to the two nearest reference curves.

FS was tested with noisy simulations generated with PROF-LCPC. Some results of FS computation with advection and radiation phase functions ADV and RAD are shown in Tab (2).

Table 2. Result of forward scattering estimation with artificial radiation and advection fogs

Phase function	$V_{\text{met-REF}}$ (m)	$V_{\text{met-EST}}$ (m)	Rel. Err.	FS
RAD	100	100.6	0.117	2.1
ADV	100	102.3	0.274	3.33

The measure FS should be seen as a classification measure that links the forward scattering of a fog to one of the reference fogs G_1 to G_4 . Here, for the radiation type droplet size distribution RAD, a value of 2.1 is found for FS , which makes it similar to G_3 in terms of forward scattering (see Fig. 7), i.e. a moderate advection fog according to Shettle & Fenn (see Fig. 3 left). The radiation type droplet size distribution RAD has an FS value of 3.3, which puts it between G_2 and G_1 , i.e. a rather moderate advection type of fog.

5.3.3 Validation with Real Fog Experiments

The simulated images generated with PROF-LCPC showed higher luminance values in advection fog than in radiation fog for equivalent values of V_{met} . As shown on Fig. 4 (right), luminance values measured in artificial fog can be ten times higher in advection fog than in radiation fog. This effect is stronger than in the simulation. This could come from the fact that we were dealing with very dense fogs.

The relative luminance of a lamp in advection fog is 4 to 10 times that of the same lamp in radiation fog for V_{met} comprised between 15 m and 45 m.

We now want to apply the method developed on synthetic luminance maps, using pairs of light sources in order to estimate k (see Eq. (5) and composing the estimations using Eq. (6) and Eq. (7). The results are shown in Tab. (3).

Table 3. Estimation of V_{met} with different types of artificial fog

Radiation fog							
$V_{met-REF}$ (m)	8	9	12	16	20	25	34
$V_{met-EST}$ (m)	6.3	6.2	8.5	9.5	12.5	15.6	37.5
Advection fog							
$V_{met-REF}$ (m)	11	15,5	22	28	33	34	43
$V_{met-EST}$ (m)	8.2	11.1	12.8	14.1	17.1	17.5	24.7

The fact that the estimated visibility is almost systematically lower than the reference visibility apparently contradicts the theory, but is explained by border effects on the transmissometer (see Fig. (2)) We can see in Tab.(3) that the estimation of V_{met} is better achieved in radiation fog than in advection fog. That was also the case with simulated images. The sensitivity composition method was applied with the six possible pairs of light sources. The mean error is about 50 % in radiation fogs, and 72 % in advection fogs. It is therefore logical that computation of the intrinsic luminance of sources using the method exposed in Sec. 5.3 leads to more error for advection fogs than for radiation fogs.

We can see in Fig. (8) that the relative error in the estimation of the intrinsic luminance of the sources is less than 100 % for radiation fogs. It can be over 1000 % for advection fogs. The computation of the measure FS using our tabulated errors as shown in Sec. 5.3 is not satisfactory. All measures give more error than the G1 fog in simulation, leading to $FS = 5$. This could come from the fact that the experiments were conducted in very dense fogs and that the tabulated function were computed with sources at different distances in simulation and in the fog chamber. The tabulated functions of relative error on the computation of the intrinsic luminance obtained from simulation seem ill suited for actual fogs. Nevertheless, the computation of a relative error on the estimation of L_0 seems to be relevant to differentiate fogs with respect to forward scattering.

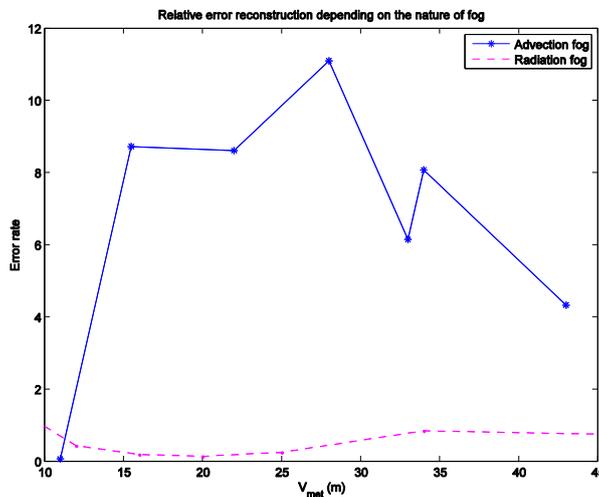


Figure 8. Error rate when estimating intrinsic luminance

5 Applications

It is a fact that drivers suffer visual impairment in fog at night, especially in dense fog conditions, and tend as a result to adopt risky behaviours. Adapting the intensity of the vehicle lighting systems may help drivers toward safer behaviours. New proposals are emerging since recent changes in European regulations. Some of those changes concern the intensity of vehicle rear lamps, which may now be adapted to the visibility conditions (UNECE, 2007), so as to maintain the visibility of the rear lights constant at some distance (Luce 2005). This involves using the meteorological visibility distance, derived from the parameter k of Beer-Lambert model, in order to compensate for the attenuation of light through fog. Solutions have been tested, using lidar technology to estimate V_{met} (Klett, 1981; Pirodda, 1997). We have shown in Sec. 3 that an observer could perceive very different luminance levels from light sources at the same distance with the same V_{met} depending on the fog droplet size distribution. This leads to the conclusion that using only Beer-Lambert model of light propagation in adaptive lighting could lead to wrong adaptation of the intensities of the lights.

We claim that granulometry must be taken into account in adaptive lighting systems for nighttime fog. Cameras or lidars used to estimate the density of fog at night should also provide a granulometry related parameter in complement to V_{met} since it is not sufficient to describe the visual effect of fog on perception (see Fig. 3 (left) and Fig. 4 (left)). We believe that recent developments in cameras (higher definition and, more importantly for our applications, higher dynamic range), could make it possible to implement the proposed method.

5 Conclusion and Outlook

We have presented a new way of characterizing meteorological visibility distance in nighttime fog with a camera that needs at least one image and three light sources with known intensity and at known distance. We showed that the method could be extended to any number of sources and that it could increase the range and confidence on the estimation of the extinction coefficient k . This method improves previous results, particularly in the case of dense fogs. But still, a bias exists that is related to the scattering of light by droplets. We showed the need for a more complete model than classic Beer-Lambert's extinction law for light propagation in fog at night.

We have proposed a measure related to the forward scattering of the fog, an aspect of light propagation in fog that is linked to fog granulometry and that strongly impacts on the appearance of light sources at night. We estimate our measure FS in reference to a tabulated function computed from simulation. The next step is to generalize this function with a functional description instead of a tabulated one and make reference to real observations through a calibration process. We showed that forward scattering should not be neglected, particularly with regard to recent evolutions in road safety transportation systems such as adaptive lighting.

This method should be extended to dynamic situations in order to be implemented in road safety applications. One possible way of achieving this goal would be to detect and follow a single light source of known distance during multiple frames with a high dynamic range camera. The methodology exposed in this paper could be applied to such cases.

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