# Template-matching based detection of hyperbolas in groundpenetrating radargrams for buried utilities 

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## 9Abstract

10Ground-Penetrating Radar (GPR) is a mature geophysical technique that is used to map utility 11pipelines buried within 1.5 m of the ground surface in the urban landscape. In this work, the 12template-matching algorithm has been originally applied to the detection and localization of 13pipe signatures in two perpendicular antenna polarizations. The processing of a GPR 14radargram is based on four main steps. The first step consists in defining a template usually 15from finite-difference time-domain simulations made of the nearby area of the hyperbola apex 16associated with a mean size object to be detected in the soil whose mean permittivity has been 17previously experimentally estimated. The second step consists in a pre-processing on the raw 18radargram to correct variations due to antennas coupling, then the template matching 19algorithm is used to detect and localize individual hyperbola signatures in an environment 20containing unwanted reflections, noise and overlapping signatures. The distance between the 21shifted template and a local zone in the radargram based on the L1 norm allows to obtain a 22map of distances. A user-defined threshold allows to select a reduced number of zones having 23a high similarity measure. In a third step, in each zone minimum or maximum discrete 24amplitudes belonging to a selected hyperbola curve are semi-automatically extracted. In the 25fourth step, the discrete hyperbola data ( $\mathrm{i}, \mathrm{j}$ ) are fitted by a parametric hyperbola modeling

26using a non linear least square criterion. The algorithm has been implemented and evaluated 27on numerical radargrams, and afterwards on experimental radargrams.

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29Keywords: Ground-penetrating radar, pipe detection, hyperfrequency, template matching, soil 30characterization, ultra-wide band

## 321. Introduction

33Ground-penetrating radar (GPR) is a well-known non destructive technique for imaging 34shallow subsurfaces by a propagating electromagnetic energy downward into the ground from 35a transmitting antenna that is reflected at subsurface boundaries between media possessing 36different electromagnetic (EM) properties and is collected by a receiving antenna. GPR is a 37proposed technology to map utility pipelines in urban environments (fiber optics, 38telecommunication lines, electrical cables, water and gas pipes, district heating network), 39most of them have been buried within 1.5 m of the ground surface [1-2]. The buried targets 40produce characteristic hyperbolic signatures in the radargram (Bscan) issued from the moving 41of the GPR system along a linear path. Hyperbolic signatures represented in the distance-time 42domain are specific to the target size, shape and dielectric characteristics and the orientation 43of the electric field [3-4]. Challenges lie in detecting and classifying targets in an environment 44with variations in the surface cover (asphalt, paving, sand, gravel, grass), in the subsoil with 45spatial vertical or horizontal variability of the soil texture (water content, backfill soil, 46multilayers...), and the presence of buried targets that can be close to each other (overlapping 47hyperbolas) and have wide variations in dimensions, and in dielectric properties (metallic or 48non-metallic).

49An ultra-wide band (UWB) ground-coupled radar operating in the frequency band [0.46;4] 50 GHz and made of bowtie slot antennas has been preferred to an air-launched radar because it 51increases energy transfer of electromagnetic radiation in the sub-surface and penetration depth 52[1,5]. Moreover, UWB GPR and SFCW (step frequency continuous wave) have been used to 53probe the soil structure using the benefit from both low and high frequencies bringing a 54compromise in terms of depth resolution and penetration in a single measurement. This paper 55is focused on the processing and analysis of radargrams using the semi-automatic template56matching algorithm applied to GPR application in order to recognize hyperbolas produced by

57buried targets and extract information from them: detection, localization, and characterization. 58Special attention has been paid to study the effects of the dielectric contrasts between the 59targets and the surrounding soil and also of the polarization configuration of the GPR system 60relative to the main axe of canonical targets (pipes or strips).

61In the literature, several works deal with the semi-automatic buried target detection and 62characterization. There are two main classes of methods: in the first class the model of the 63pattern is given a priori or designed by hand, in the second class the model is learned from the 64collected data. The methods based on Hough transform and its derivatives [6-8], genetic 65algorithm (GA) [10], and image segmentation and hyperbola fitting [16] are in the first class. 66The methods based on artificial neural networks (ANNs) [8-11], support vector machine 67(SVM) [12], and wavelet analysis [13-15] are in the second class. The template matching [176820] is widely used in pattern recognition, since it is a simple method, based on a prior model 69extraction from one or several images, which leads to a good accuracy for detecting targets 70with reduced variability in observed images. Surprisingly, this approach has not been 71previously used in GPR, when observed patterns due to objects of interest are of quite similar 72aspects in the radargrams.

73The template-matching algorithm is readily implemented on a computer and has a reasonable 74calculation time providing that the image and the template sizes are not too large. The 75template does not need to contain a hyperbola response very similar to the one to be detected, 76it only needs to be able to discriminate between hyperbolas of interest and the background 77(noise and small heterogeneities in the soil compared to the targets). The analysis of the 78distance map based on the L1 norm using an optimal threshold has allowed to select a limited 79number of template positions. In this work, the use of two antenna orientations along a profile 80(named polarization diversity) has allowed to obtain two distance maps from which a mean 81distance map should improve hyperbola detection in one of the two radargrams where the

82polarized response appears too weak and thus not detectable. The fitting of each hyperbola 83applied on maximum or minimum amplitudes is performed by a parametric analytical model 84based on the straight ray path hypothesis and using the constraint least square (LS) criterion. 85Considering canonical objects (pipes and strips), the template-matching algorithm has been 86tested on simulated (commercial software Empire) and experimental radargrams.

87This paper is structured as follows: in section 2, the radar geometry is briefly described that 88was modeled using FDTD simulations. The parametric analytical ray path modeling was used 89to model hyperbolas. Section 3 deals with the steps of the pattern recognition algorithm that 90includes mainly the template-matching method and the parametric hyperbola fitting. The 91developments and validation were first performed on synthetic images. In section 4, a few 92experimental images acquired in a laboratory site are analyzed and the results associated with 93the parameter analysis are discussed. Finally, in section 5 conclusions and perspectives are 94drawn.

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## 962. Modeling of the GPR system

## 97A. Numerical FDTD modeling

98The GPR SFCW system made of a pair of shielded bowtie slot antennas designed on a FR4 99substrate, with real relative permittivity $\varepsilon^{\prime}=4.4$ and thickness $\quad \varrho=1.6 \mathrm{~mm}$ (see Fig. 1a), has 100been preliminary studied using FDTD simulations (commercial software Empire) [5]. The 101antenna radiation characteristics are adequate for operating in an UWB in the frequency range $102[0.46 ; 4] \mathrm{GHz}$ very close to the soil surface (ground-coupled). Each antenna is enclosed in a 103shielded conductive rectangular box filled with a three-layered absorbing foam with 104dimensions $362 \times 231 \times 67.5 \mathrm{~mm}^{3}$. Because the simulation time of the pair of antennas moving

105on the soil surface on a distance close to 1.6 m appears prohibitive for a parametric study 106(more than one week using a CPU i7-950 3.07 GHz ), a pair of more simple planar antennas, 107named blade dipoles working in the same frequency band, but with a higher reflection 108coefficient, has been used. The blade dipoles designed on a FR4 substrate are non shielded 109and their dimensions are $290 \times 56 \mathrm{~mm}^{2} \quad$ (see Fig. 1c) [21, 22]. 110The offset between antennas in simulations and experiments has been fixed to 60 mm , and the 111elevation $h_{s}$ above the soil is equal to 10 mm . The soil electrical parameters ( $\varepsilon_{s}^{\prime}, \sigma_{s}$ ) are 112assumed constant across the frequency range. The GPR system is moved linearly on the soil 113surface with a step $\Delta y=40 \mathrm{~mm}$ (see Fig. 1a) to acquire a radargram. Because the 114experimental and modeled antennas appear as complementary UWB dipoles, the polarization 115of the electric field $\left(E_{\theta}, E_{\phi}\right)$ appears perpendicular. In the simulations, the excitation current 116has the shape of the derivative of the Gaussian function with a time zero estimated to 0.3 ns 117and a duration ( $99 \%$ of the total energy) of 0.5 ns (peak frequency 1 GHz , bandwidth 3 GHz ). 118

119B. Analytical ray path modeling

120The analytical modeling based on the ray path hypothesis supposes that the target is localized 121in the far-field zone of the GPR system. Such a modeling helps the interpretation of the 122earliest reflection component of a hyperbola pattern. Considering a buried canonical object, 123 such as a cylindrical pipe, the linear displacement of the GPR system on the soil surface ( $\mathrm{h}=0$ ) 124such as depicted in Fig. 1a gives a time-distance curve with the shape of a hyperbola. The 125two-way (round-trip) travel time can be expressed as a function of the horizontal position $126 y$ of the radar, the radius $R$, its horizontal location at $y_{0}$ and its depth $d$ under the 127soil surface [16]. The lateral center-to-center antenna distance $S R$ is considered in this

128study because of the marked dimension size of the antennas in the two main polarization 129configurations．

130The equations associated with the travel－time write as follows：

$$
\begin{align*}
& y_{T}=y_{i}-S R / z \\
& y_{R}=y_{i}+S R / 2 \\
& i z i  \tag{1}\\
& i z
\end{align*}
$$

$$
\begin{aligned}
& T_{T \times 2 t \operatorname{arget}}=\left[\left(y_{0}-y_{T}\right)^{2}+(d+R)^{2}\right]^{0.5}-R \\
& T_{t \operatorname{arget} 2 R x}=\left[\left(y_{0}-y_{R}\right)^{2}+(d+R)^{2}\right]^{0.5}-R
\end{aligned}
$$

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133The velocity $v$ of the medium depends on the real relative permittivity $\varepsilon_{s}$ such as：
\}\} $<\varepsilon \operatorname{rSub}\{\operatorname{size} 8\{$ s $\}\} \operatorname{rSup}\{\operatorname{size} 8\{\prime\}\}$ ）$\}\{$
134
$v=c / \sqrt{\varepsilon_{s}^{i}}\left(\varepsilon_{s}^{i}\right.$
${ }^{135}$ The generalized hyperbola equation $\left(y_{i}, t_{i}\right)$ including the antenna offset is expressed by：

136

$$
\begin{equation*}
t_{i}=\left(T_{T \times 2 \text { araget et }}+T_{\text {target2Rx }}\right) / v \tag{3}
\end{equation*}
$$

137Thus，the hyperbola depends on five parameters（ $S R, y_{0}, d, R, v$ ）．
138A preliminary parameter study presented in Fig． 2 was conducted considering two 139configurations of the SFCW GPR system：parallel（SR＝291 mm），and end－fire（SR＝422 mm） 140［5］．Assuming a soil with $\varepsilon_{\mathrm{s}}^{\prime}=3.5 \quad\left(\sigma_{\mathrm{s}}=0\right)$ and a buried pipe at depth $\mathrm{d}=180 \mathrm{~mm}$ with several 141radius values $[0,40,80,120,160] \mathrm{mm}$ ，we first remark from Fig．2a that when $\mathrm{SR}=0$ the radius 142reduces the arrival time as if the velocity was increased；when $\mathrm{R}=0$ ，a time difference of 0.64 $143 n$ at the apex is observed between the configurations $\mathrm{SR}=0$ and $\mathrm{SR}=291 \mathrm{~mm}$ ．We observe that 144the antenna distance SR flattens the apex and produces a delay in the arrival time，particularly 145significant when SR＝422 mm（see Fig．2b）．The time difference at the apex increases with the 146soil permittivity as visualized in Fig．2c．In the configuration $S R=291 \mathrm{~mm}$ ，we remark that the 147arrival time difference at the apex with a pipe radius less than 40 mm appears less than 0.1 ns

148(at the apex when $\mathrm{R}=0$, if $\mathrm{SR}=291 \mathrm{~mm} \mathrm{t} 0=2.89 \mathrm{~ns}$, and if $\mathrm{SR}=422 \mathrm{~mm} \mathrm{t} 0=3.45 \mathrm{~ns}$ ); thus, in 149this range the radius value cannot be distinguished in a radargram and particularly when 150 considering a time step of $\Delta t=5.56 \mathrm{E}-11 \mathrm{~s}$ used in experiments. With a larger antenna 151distance $\mathrm{SR}=422 \mathrm{~mm}$, the radius variations appear more easily separable at the apex, however 152the hyperbola apex appears more flat and its amplitude will appear lower when the arrival 153time is higher. The variation of the soil real relative permittivity in the range [4;10] 154(R=40 mm) and illustrated in Fig 3c highlights that the hyperbola slope increases with the 155permittivity value, and the apex appears sharper.

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1573. Template matching technique and hyperbola fitting

158The proposed algorithm based on template-matching aims to detect semi-automatically 159hyperbola signatures without a preliminary training period, and to estimate target 160 characteristics as reliably as possible.

161The radargram processing is made of a series of steps:
162- A pre-processing performed on the raw radargram to remove the antenna direct coupling and 163the clutter removal to enhance radargram quality;

164- The construction of an amplitude distance map based on the translation on the radargram of 165a predefined template at every possible positions where a mean amplitude distance is 166evaluated according to the $\mathrm{L}_{1}$ norm [8-9]. A threshold value allows to select local discrete 167minima on the distance map that corresponds to a high level of similarly with the template.

168- At the selected positions, the hyperbola points close to the apex that can correspond to a 169maximum or a minimum amplitude are extracted. For each curve, a fitting of the points is 170performed using a parametric model;

171- The estimation of the parameters describing the hyperbola curve such as $d, R, v$ is obtained 172using a hyperbola fitting of the points to the analytical relation (3) according to the LS

173criterion. The Hessian matrix in 2D and particularly its eigenvalues can be used to 174characterize the uncertainties on the estimated parameters $(d, v)$. 175

176A. Pre-processing

177In a radargram, the strongest and first signal detected associated with antenna ground178coupling may mask the hyperbolas of shallow buried objects. Depending on the depth of the 179objects and more precisely the degree of interaction of their responses with the signal of the 180clutter, an adequate clutter reduction technique has to be chosen [1, 22]. In this study, we have 181considered objects buried at a depth higher than 10 cm , thus the median subtraction at each 182time sample is assumed sufficient to reduce the horizontal component of the clutter. 183Preliminary to image processing, the duration of the radargrams considered has to be limited 184(10 ns for numerical data, and 5 ns for experimental data) in order to eliminate the soil 185background without potential targets. Moreover, the distance-time units ( $\Delta y=40 \mathrm{~mm}$ in 186numerical and experimental data) of the template image defined have been scaled according 187to the image to be analyzed that supposes to perform a time interpolation; as in synthetic 188radargrams the time step was fixed to $\Delta t=2.39610^{-11} \mathrm{~s}$ and $1.79110^{-11} \mathrm{~s}$ in the parallel and 189end-fire configurations, respectively (see Fig. 3a and 3b), in experiments it was fixed to $1905.5610^{-11} \mathrm{~s}$. A time zero correction has been applied in each radargram according to the 191shape of the excitation signal (the first positive amplitude component arrives here at 0.5 ns ) to 192obtain a time zero at the wave ground bounce.

193It must be underlined that in a radargram the data representing discrete amplitudes can be 194either binary or signed, depending on the quality and the dynamic range before the 195conversion; as in simulated data there is no background noise or unwanted reflections, the

196conversion of signed amplitudes into binary amplitudes doesn't change the image quality. In 197experimental data, the signed amplitudes will be preferred.

199B. Template-matching technique

200The template-matching algorithm supposes to preliminary define a template including the 201average aspect of the different hyperbolas of interest in the radargrams to be analyzed. A 202preliminary study involving buried pipes under study was made to visualize and interpret a 203few synthetic hyperbola signatures. This study is an extension of the work in [3,23]. Synthetic 204radargrams have been obtained from FDTD simulations (software Empire) considering a 205system with blade dipoles (offset=60 mm, $\mathrm{h}_{\mathrm{s}}=10 \mathrm{~mm}$ ) in both configurations, parallel and end206 fire (see Fig. 3a and 3b) moving linearly on a homogeneous soil ( $\varepsilon_{s}^{\prime}=3.5, \sigma_{s}=0.01$ S.m ${ }^{-1}$, $207 \mathrm{~h}=1 \mathrm{~cm}$ ). A pipe (radius $\mathrm{R}=32 \mathrm{~mm}$ ) with fundamental dielectric characteristics such as air208filled, conductive, and dielectric ( $\varepsilon=15$ ) was buried at depth $\mathrm{d}=168 \mathrm{~mm}$ [3-4]; the depth 209 d is measured between the soil surface and the top surface of a pipe. The hyperbola 210signatures visualized in Fig. 4a, 4b, 4c, 4d, 5a 5b and 5c are plotted in the time range 8 ns , and 211the time vectors were interpolated to match the experimental time step $\Delta t=5.5610^{-11} \mathrm{~s}$. 212In general, we remark that the parallel configuration such as presented in 4 a and 4 b 213(SR=116 mm) generates multiple reflections with higher amplitudes in the hyperbola legs 214whatever the dielectric characteristics of the pipe as compared to the end-fire case. These 215reflections correspond to wave bounces between the soil surface and the pipe top. In the case 216of a non conductive pipe, waves propagate in the pipe, and we observe a marked reflection at 217the bottom of the pipe when it is filled with a medium having a high contrast with the soil, in 218particular in water, that induces a wave velocity difference. Moreover, the amplitude of the 219backscattering waves appears reduced when the soil is attenuating.

220In the end-fire configuration, such as visualized in Fig. 4c and 4d (SR=340 mm), hyperbola 221patterns have few components, and appear more compact. The conductive pipe induces a 222specific inverse polarity (negative amplitude) in the reflection pattern that allows to 223distinguish it. In both polarizations, we remark that the conductive pipe has the strongest 224reflection amplitude. In the case of a high dielectric pipe ( $\varepsilon=15$ ), Fig. 5a and 5b highlight 225that in both polarizations multiple reflections induced between the top and bottom of the pipe 226can be distinguished because the velocity is relatively low inside the pipe; the parallel 227polarization gives a higher amplitude response. In Fig. 5b, artifacts that are induced by the 228clutter removal treatment appear visible because in the present case the clutter component and 229the target signature overlap (see Fig. 5c). Samples of these synthetic pipe signatures can serve 230as template images in the experiments.

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232The template does not need to be perfect, it only needs to allow us to discriminate between 233hyperbolas of interest and the background (small heterogeneities). When the set of hyperbolas 234of interest is too large to be represented by a single pattern, several patterns can be used in 235sequence. Each defined template includes a small portion of a hyperbola pattern in the vicinity 236 of the apex and is scaled in time, distance and amplitude to the radargram under analysis.

237Assuming an observed image $g(i, j)$ with a size $M \times N$, and a template image $t(i, j)$, 238both with scaled amplitudes, we define the $\mathrm{L}_{1}$ norm distance map $\quad E$ between $g$ and $t$ by the 239following relation [18]:

$$
\begin{equation*}
E(m, n)=\sum_{i} \sum_{j}|t(i, j)-g(i-m, j-n)| \tag{4}
\end{equation*}
$$

241The summation is evaluated at all pixels $(\mathrm{i}, \mathrm{j})$ of the template $t$ that is translated to all possible 242positions (m,n) along the observed image $g$. This leads to an image of L1 distances, also 243named absolute distance map. The position (m,n) at which the smallest value $E(m, n)$ is

244obtained corresponds to the best match between the template $t$ and the corresponding sub245image in $g$. A threshold value allows to select a limited number of local minima 246corresponding to distances $E$ less than the threshold where the template is well matched 247with enough amplitude (visualized by " + " signs on the radargram, see Fig. 7c and 8c).

248The template-matching algorithm has been first applied on synthetic data. The GPR system 249made of blade dipoles in the parallel (see Fig. 6) and end-fire configurations has been 250considered on a homogeneous soil structure including three close buried pipes at several 251depths. The pipes, from the left to the right, made of conductor ( $n^{\circ} 1$ ), dielectric ( $n^{\circ} 2, \varepsilon_{s}^{\prime}=9$ 252) and air ( $n^{\circ} 3, \varepsilon_{s}^{\prime}=1$ ) are separated by a distance equal to 300 and 250 mm respectively. The 253pipe radii are respectively 32,22 and 11 mm . The radargrams after clutter removal are 254presented in Fig. 7c and 8c in the parallel and end-fire configurations (time range 10 ns ). The 255comparison of both radargrams highlights a difference in shape and amplitude in the 256hyperbola signatures of the three objects that is explained by excitation direction of the 257electric field towards the pipes which are either conductive or dielectric. In both polarizations, 258we observe that the air-filled pipe has a significant lower response than the two other pipes, 259and thus appears masked; this phenomenon is caused by the weak permittivity contrast 260between the pipe and the soil that will be higher in a real soil.

261In the end fire configuration, the synthetic template ( $189 \times 7$ pixels, $\Delta t=1.79110^{-11} \mathrm{~s}$ ) 262visualized in Fig. 8a represents the upper part, near the apex, of the hyperbola signature of the 263conductive pipe in the end-fire configuration to further consider hyperbolas that overlap 264because of close objects; the apex appears in the middle of the time range. The 265complementary version of this template ( $141 \times 7$ pixels, $\Delta t=2.39610^{-11} \mathrm{~s}$ ) has been used in 266the parallel configuration as it appears more adapted. At present, the template includes a

267significant upper zone without signal to allow the detection of first hyperbolas containing 268information.

269In the parallel configuration, the similarity measure, applied on Fig. 7c (considering a time 270range 8 ns to reduce late reflections) while translating the template of Fig. 7a, leads to the 271distance map of Fig. 7b. In this last figure, two marked minima can be visualized that 272correspond to the positions of both significant hyperbolas. Defining the maximum threshold 273value to 120, the first two local positions of hyperbolas are detected, as visualized in Fig. 7c 274(time range 10 ns ). We remark that the air-filled pipe has not been detected. In the end-fire 275configuration, the analysis of the radargram presented in Fig. 8c using the template of Fig. 8a 276according to the template matching algorithm, gives the distance map of Fig. 8b. The 277maximum threshold value of 162 allows to detect the two first hyperbola zones associated 278with the conductive and dielectric pipes such as visualized in Fig. 8c (time range 10 ns). As 279previously, the pipe air-filled pipe cannot be detected.

280The polarization diversity (joint information from both perpendicular polarizations) can be 281used to enhance the detection in each polarization [23]. Thus, a mean distance map can be 282calculated from the individual distance maps in the parallel and end-fire configurations. A 283validation with experimental data of this point is described in Section 4.

284
285C. Hyperbola fitting

286For each detected template location, the next processing step consists in three sub-steps: 287location of the maximum or minimum amplitude arch associated with first arrival times, 288extraction of the discrete amplitudes, and fitting of the arch by an analytical hyperbola 289equation.

290The curve points associated with the first arrival times of a hyperbola may correspond to a 291maximum or a minimum amplitude. Because higher order reflections in a hyperbola pattern

292may produce a stronger amplitude as compared to the amplitude of the first reflection, an user 293interaction is necessary (semi-automatic) to select a hyperbola curve either on the upper or on 294the lower half zone of the template position. Starting from the middle point of the template, 295close points belonging to the hyperbola curve on the left and on the right legs (usually 3 or 4 296points) are extracted step-by-step. It is possible to extract points outside the template location 297mainly when no signature overlaps. Because the number of curve points extracted is limited, a 298polynomial fitting of the second order was made to refine the estimate of the abscissa $y_{0}$ at 299the apex. Then, knowing a priori parameters SR and $y_{0}$, a fitting with the analytical 300modeling described by equation (3) and based on a constraint LS criterion has allowed to 301estimate the three parameters $|d, R, v|$. The constraint consists in defining for each parameter 302a minimum value, such as in the present case 40 mm (or 30 mm ), 50 mm and $70 \mathrm{~mm} . \mathrm{ns}^{-1}$ for $303 \mathrm{R}, \mathrm{d}$ and v , respectively and a maximum value $60 \mathrm{~mm}, 200 \mathrm{~mm}$ and $190 \mathrm{~mm} . \mathrm{ns}^{-1}$, respectively. 304In the analysis of the numerical radargrams, initial values are respectively $40 \mathrm{~mm}, 100 \mathrm{~mm}$ 305and $134.16 \mathrm{~mm} . \mathrm{ns}^{-1}$ (for $\varepsilon^{\prime}=5$ ). In order to confirm the stability of the algorithm, a second 306fitting in a similar manner based on the analytical formula is performed using the last 307estimated parameters of R, d and vas initial values. Each hyperbola fitting in the radargram is 308made independently of each other.

309Considering the radargrams in Fig. 7c and 8c associated with the parallel and end fire 310configurations, the parameter values obtained from the hyperbola fitting are collected in 311Tables 1 and 2, respectively. In each template location, the hyperbolas fitted were plotted (see 312Fig. 7c and 8c). From the parameter values, we remark that the pipe radius cannot be properly 313estimated, as it appears smaller than the radiating aperture of an antenna (on the order of the 314first Fresnel zone in the soil [24]); thus, the radius estimate remains equal to the initial value 315defined in the optimization function. In general, the real permittivity value was estimated with

316an uncertainty less than $15 \%$, and this uncertainty influences the estimation of the depth. We 317observe that the curve fitting of pipe $\mathrm{n}^{\circ} 2$ gives a lower permittivity estimate in both 318 configurations as compared to the curve fitting of pipe $n^{\circ} 1$. In the end-fire configuration, we 319remark from Fig. 8c that the pipe signature $n^{\circ} 2$ is made of a significant second reflection that 320appears quite higher that the first reflection, that makes it difficult to detect. Consequently, the 321second hyperbola signature which arrives 0.4 ns later has been analyzed and the depth will 322need an adjustment of -69.3 mm (see Table 2). The fitting of this signature gives a higher 323objective function fval for pipe $n^{\circ} 2$ because 4 points has been considered in the left leg of the 324hyperbola. Otherwise, the objective functions are of the order of $10^{-3}$. In general, we observe 325that the uncertainty of the depth is less than $22 \%$.

326

|  | d | R | ع' | $\mathrm{t}_{0}$ | $\mathrm{X}_{0}$ | fval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Pipe } n^{\circ} 1 \\ \left(\left(^{\text {st }}\right.\right. \\ \text { maximum }) \\ \hline \end{gathered}$ | 184.3 mm | 40 | 3.25 | 2.3 ns | $\begin{gathered} -189.1 \\ \mathrm{~mm} \end{gathered}$ | $1.6810^{-3}$ |
| $\begin{gathered} \text { Pipe } n^{\circ} 1 \\ \text { (true } \\ \text { values) } \\ \hline \end{gathered}$ | 168 mm | 32 | 3.5 |  | -185 mm |  |
| $\begin{gathered} \text { Pipe nº } \\ \left(\left(^{\text {st }}\right.\right. \\ \text { maximum }) \\ \hline \end{gathered}$ | 98.8 mm | 40 | 2.92 | 1.26 ns | 113.1 mm | $7.6710^{-4}$ |
| $\begin{gathered} \hline \text { Pipe }{ }^{\circ}{ }^{\circ} \\ \text { (true } \\ \text { values) } \\ \hline \end{gathered}$ | 79 mm | 20 mm | 3.5 |  | 115 mm |  |

327 Table 1: Parameter estimation in the parallel configuration for the synthetic Bscan of Fig. 7c

|  | d | R | $\varepsilon^{\prime}$ | $\mathrm{t}_{0}$ | $\mathrm{x}_{0}$ | fval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pipe n${ }^{\circ} 1$ <br> $\left(1^{\text {st }}\right.$ <br> minimum $)$ | 150 mm | 30 | 3.93 | 2.94 ns | -188.3 | $8.2310^{-3}$ |
| Pipe n |  |  |  |  |  |  |
| (2 $2^{\text {nd }}$ <br> minimum $)$ <br> $\left(1^{\text {st }} \begin{array}{c}\text { minimum } \\ \text { by } \\ \text { extrapolation } \\ )\end{array}\right.$ | 80.6 mm | 149.9 mm | 30 | 2.99 | 2.55 ns | 113.9 mm |
| mm | $1.6710^{-2}$ |  |  |  |  |  |

328 Table 2: Parameter estimation in the end-fire configuration for the synthetic Bscan of Fig. 8c
3304. Evaluation on experimental data

331Targets with different dielectric characteristics have been buried in a large sandy box of the 332public square Perichaux, Paris $15^{\text {th }}$ district. The sand was not compacted and its relative 333dielectric permittivity was estimated to 3.5 from CMP (Common Mid Point) measurements. 334The time zero was corrected according to CMP measurements that have allowed to locate the 335direct waves. The pair of bowtie slot antennas (see Fig. 1b) separated by a 60 mm offset was 336moved linearly on the soil surface with a step of 40 mm to acquire a radargram. Both 337polarizations, parallel and end-fire, have been considered separately.

338The complex transmission coefficient $\tilde{S}_{21}$ at the receiving antenna port was measured and 339stored using a vector network analyzer (VNA) ANRITSU MS 2026B in the frequency range $340[0.05 ; 4] \mathrm{GHz}$ ( 1601 samples). To obtain the transmitted signal in the time domain, an 341apodization (zero padding) has been applied to $\tilde{S}_{21}$ in order to smoothly extend the 342 frequency band from 4 to 9 GHz . The excitation signal used in FDTD simulations, the first 343derivative of the Gaussian function with a duration equal to 0.5 ns , has been introduced in 344each experimental signal in the frequency domain. The product of the spectrum of the 345 excitation signal (convolution in the time domain) with the measured $\tilde{S}_{21}$ has been 346performed to further calculate the inverse Fourier transform (IFFT) and obtain time data.

347In the sandy box, a 25 mm diameter dielectric pipe and a thin vertical 10 mm width ( 2 mm 348thick) conductive strip were buried at depths estimated to 160 and 170 mm respectively such 349as presented in Fig. 9. Both objects are separated by a 750 mm distance. Firstly, a synthetic 350template was computed from 3D FDTD simulations (software Empire) considering the 351detailed bowtie slot antenna geometries (section 2.A) and 32 mm radius conductive pipe 352buried in a soil with a real relative permittivity $\quad \varepsilon^{\prime}=3.5 \quad\left(\quad \sigma=10^{-2} S . m^{-1}\right)$. In this template

353visualized in Fig. 10, it was wise to define a compact hyperbola signature with reduced 354multiple reflections to further detect different hyperbolas in a radargram in both polarizations. 355Thus, a conductive pipe in the end-fire configuration has been considered. The template (see 356Fig. 10) was scaled in time and amplitude to match the experimental time step $357 \Delta t=5.56 E-11 \mathrm{~s}$ (step distance 40 mm also used in experiments), and the amplitude range of 358the radargrams presented Fig. 11b and 11d (time range 5 ns ). Signed amplitudes have been 359used here to not deteriorate the image quality.

360The template-matching algorithm was performed on the experimental radargrams visualized 361in Fig. 11b and 11d and associated with both parallel and end-fire polarizations. The L1 norm 362distance maps are presented in Fig. 11a and 11c. The maximum threshold values leading to 363the detection of the first and most significant hyperbolas are 0.162 and 0.195 respectively; 364higher values give additional detections (false alarms) that don't correspond to buried objects 365but to background heterogeneities. We remark that the end-fire polarization does not permit to 366detect the air-filled pipe. The results of the parameter evaluation from the LS fitting of each 367hyperbola detected are collected in Tables 3 and 4. In general, the positions $y_{0}$ of the objects 368appear correctly evaluated. Concerning the real permittivity value of the soil, the parallel 369configuration gives higher estimates as compared to the end-fire configuration, and 370consequently the target depths appear more important; from Table 3, we remark that the 371depths of the pipe and the strip have been both evaluated to 200 mm , and more important than 372those a priori evaluated (see Fig. 9). In the case of a soil having weak permittivity variations, 373an additional step would be to find an optimum permittivity value issued from the several 374estimates. We remark that the objective function fval associated with the LS fitting is 375presently higher for experimental data than for synthetic data, of the order of $10^{-2}$, because the 376image quality appears lower.

377To detect the air-filled pipe in Fig. 11d, joint information from radargrams issued from the 378parallel and end-fire configurations (polarization diversity) could be used. Thus, the mean 379distance map was calculated from the individual distance maps in the parallel and end-fire 380configurations (see Fig. 11a and 11c) leading to the result visualized in Fig. 12a. A weak 381threshold (0.18) situated between the previous ones ( 0.162 and 0.195 ) associated with both 382polarizations, has allowed here to detect both hyperbolas without false alarms in the end-fire 383configuration, that was not possible when this polarization was only considered.

384Further insight into the solutions of the parameters estimated in the fitting has been gained by 385calculating the Hessian matrix H at the stationary point to evaluate its nature using its 386eigenvalues.

387The rate of convergence and sensitivity to round-off errors is given by the condition number 388of matrix H , that is the ratio of its largest to its smallest eigenvalues. In the present examples, 389fixing the pipe radius that cannot be evaluated properly has led to a decrease of the condition 390number. The eigenvalues associated with the several fitting in both polarizations are collected 391in Table 5. In general, the condition number is high, that means that correlation may exist 392between the two parameters and thus the convergence of the estimation algorithm appears 393slow.

394

|  | d | R | $\varepsilon^{\prime}$ | $\mathrm{t}_{0}$ | $\mathrm{y}_{0}$ | fval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pipe $\mathrm{n}^{\circ} 1$ <br> (maximum <br> ) | 200 mm | 60 | 3.6 | 3.08 ns | 458.9 | $2.6810^{-2}$ |
| Pipe $\mathrm{n}^{\circ} 1$ <br> (true <br> values) | $\sim 160 \mathrm{~mm}$ | 12.5 mm | $3.5-4$ |  | $\sim 500 \mathrm{~mm}$ |  |
| Strip n${ }^{\circ}$ 2 <br> (maximum <br> ) | 200 mm | 60 | 3.47 | 2.99 ns | 1232 mm | $2.2310^{-2}$ |
| Strip $n^{\circ} 2$ <br> (true <br> values) | $\sim 170 \mathrm{~mm}$ | 5 mm | $3.5-4$ |  | $\sim 1200$ |  |

395 Table 3: Parameter estimation in the parallel configuration for the experimental Bscan of Fig. 396 11b

|  | d | R | $\varepsilon^{\prime}$ | $\mathrm{t}_{0}$ | $\mathrm{y}_{0}$ | fval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Strip n <br> (maximum <br> ) | 165.6 mm | 25.5 | 2.65 | 2.82 ns | 1250 mm | $1.8510^{-2}$ |

397 Table 4: Parameter estimation in the end-fire configuration for the experimental Bscan of Fig.

399

| Configuration <br> s | Eigenvalues <br> for (d;v) |
| :---: | :---: |
| Parallel | $\left(4.110^{-5} ;\right.$ |
| Pipe $\mathrm{n}^{\circ} 1$ | $\left.2.110^{-2}\right)$ |
|  | Strip $\mathrm{n}^{\circ} 2$ |
|  | $\left(4.210^{-5} ;\right.$ |
|  | $\left.1.910^{-2}\right)$ |
| End-fire |  |
| Strip $\mathrm{n}^{\circ} 2$ | $\left(4.010^{-5} ;\right.$ |
|  | $\left.9.610^{-3}\right)$ |

400Table 5: Eigenvalues of the Hessian matrix at the estimates of the depth d and velocity v from 401hyperbola fitting in both polarizations

402
403A second soil structure built in the sandy box includes two buried objects, a 25 mm diameter 404water-filled pipe and a horizontal 2 mm width strip with a 10 mm height such as visualized in 405Fig. 13a. In the parallel configuration, the radargram not presented here shows two very 406similar weak hyperbola signatures quite difficult to detect. In the end-fire configuration, the 407radargram presented in Fig. 13c shows two different hyperbola signatures. Using the template 408of Fig. 10, the template-matching algorithm has permitted to localize only the strip signature 409(threshold value 0.202 ) because the water filled pipe shows a quite different and specific 410hyperbola pattern; a threshold value higher than 0.202 will lead to detect several signatures in 411the background. The parameter estimations issued from the hyperbola fitting lead to the

412values collected in Table 6. The permittivity estimate appears close to the value issued from 413the previous experiments in the end-fire configuration.

414

|  | d | R | $\varepsilon^{\prime}$ | $\mathrm{t}_{0}$ | $\mathrm{y}_{0}$ | fval |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Strip n$^{\circ} 2$ <br> $\left(\begin{array}{c}\text { maximum } \\ )\end{array}\right.$ | 187.8 mm | 60 | 2.49 | 2.61 ns | 1299 mm | $1.0710^{-1}$ |

415Table 6: Parameter estimation in the end fire configuration for the experimental Bscan of Fig.
416
13c
4175. Conclusion

418This study is focused on the feasibility of detecting the location, depth, lateral dimension of 419long but thin buried objects such as pipes or strips in a soil using a GPR system. The most 420difficult tasks in the analysis of GPR radargrams is the split of overlapping signatures, the 421detection of a hyperbola pattern in a noisy background and in a weak image quality, the 422detection of an object with a small lateral dimension and a weak contrast with a perturbed 423surrounding medium. The work presented was carried out in two main steps: firstly, a 424theoretical parameter study (based analytical and numerical results) has consisted in analyzing 425hyperbola signatures of different dielectric pipes as a function the dielectric contrast with the 426surrounding medium, the polarization of the electric field and their diameter, and secondly a 427semi-automatic algorithm based on the template-matching technique and LS hyperbola fitting 428was developed to detect, localize and characterize a given but not restrictive hyperbola 429pattern. The template-matching algorithm does not need a training period, but needs the help 430of the user because of the great diversity of hyperbola signatures and the objective of the 431algorithm is to operate with a non perfect template.

432The template-algorithm validated on numerical radargrams issued from FDTD simulations 433has shown that distinct templates have to be associated with both polarizations. Depending of 434the level of details required, several templates may be successively used to identify

435progressively the dielectric nature of a buried target among several in a radargram. A template 436has to consider the lateral dimension of the GPR system when it is quite larger than the size of 437the object to be detected. A sensitivity analysis has to be perform to study in details the 438influence of the soil permittivity used in the template to detect hyperbolas in a radargram. The 439advantage of using a SFCW GPR is that the excitation signal used in the simulated template 440can be used in the IFFT transform. Even with a few extracted hyperbola points, the present 441work shows that main parameters associated with the target can be estimated, but may be 442slightly correlated. This correlation can be diagnosed by means of the Hessian matrix. 443Considering measurements in a controlled environment, this work has shown that synthetic 444templates can be used to analyze radargrams provided that the templates have the same time 445step and amplitude range adjusted if necessary by interpolation; thus, a set of patterns has to 446be collected. The analysis of an experimental radargram in the end-fire configuration has 447shown that in the case of a buried water-filled pipe and a vertical conductive strip, the selected 448synthetic template was not able to detect the signature of the water-filled pipe because of its 449specific signature. The template-matching algorithm applied to GPR was extended by using 450the benefit of polarization diversity. This implies to define a mean distance map using the 451distance maps of both polarizations to improve the detection of weak hyperbola signatures in 452a polarization and strengthen the algorithm robustness. Further studies will be focused on 453bringing improvements to the template-matching algorithm, particularly during its validation 454on experimental radargrams acquired in the laboratory site Sense-City [26] where different 455targets representing utilities and cracks were buried. Thus, we are planning to build a data 456base of templates with variable sizes and variable dielectric contrasts of pipes and strips and 457variable surrounding materials. This new site is a good opportunity to refine the clutter 458removal procedure and to evaluate the performances of the template matching algorithm in a 459situation close to a real environment.

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524
525

526

## 527LIST OF FIGURES

528
529Figure 1: (a) Scheme of the GPR system imaging a subsurface with a buried pipe; (b) 530Geometry of the bowtie slot antenna used in the measurements; (c) Geometry of the blade 531dipole used in the simulations

532Figure 2: Results of the parameter study performed using the ray-path modeling: variation of 533the pipe radius ( $\varepsilon_{s^{\prime}}=3.5$; $\mathrm{d}=180 \mathrm{~mm} ; \mathrm{y} 0=500 \mathrm{~mm}$ ) for antenna distances (a) $\mathrm{SR}=0$ and 291 534mm, and (b) SR=291 and 422 mm ; (c) variation of the soil permittivity considering $\mathrm{SR}=291$ 535and $422 \mathrm{~mm}(\mathrm{~d}=180 \mathrm{~mm} ; \mathrm{R}=40 \mathrm{~mm} ; \mathrm{y} 0=500 \mathrm{~mm})$

536Figure 3: Geometry of the GPR system used in FDTD simulations considering of a pair of 537blade dipoles moving on a soil with a buried pipe ( $\mathrm{d}=168 \mathrm{~mm}, \mathrm{R}=32 \mathrm{~mm}$ ) in the two 538polarizations: (a) parallel, and (b) end-fire

539Figure 4: Synthetic radargrams (after clutter removal) obtained from FDTD simulations using 540a pair of blade dipoles (offset 60 mm ) on a soil ( $\varepsilon_{s}^{\prime}=3.5$, $\sigma_{s}=0.01$ S.m $\mathrm{m}^{-1}, \mathrm{~h}_{\mathrm{s}}=10 \mathrm{~mm}$ ) 541including : (a, b) a buried air-filled dielectric pipe in the parallel and end-fire configurations 542respectively, and (b, c) a buried conductive pipe in the parallel and end-fire configurations 543respectively

544Figure 5: Synthetic radargrams (after clutter removal) obtained from FDTD simulations using 545a pair of blade dipoles (offset 60 mm ) on a soil ( $\varepsilon_{\mathrm{s}}^{\prime}=3.5, \sigma_{\mathrm{s}}=0.01$ S.m ${ }^{-1}, \mathrm{~h}_{\mathrm{s}}=10 \mathrm{~mm}$ ) 546including a buried dielectric pipe ( $\varepsilon=15$ ) in the (a) parallel and (b) end-fire configurations 547; (c) end-fire configuration without clutter removal

548Figure 6: Geometry of the GPR system, used in FDTD simulations, considering of a pair of 549blade dipoles moving on a soil including 3 different buried pipes ( $\mathrm{n}^{\circ} 1$ metal, $\mathrm{n}^{\circ} 2$ dielectric 550with $\quad \varepsilon^{\prime}=9$, and $\mathrm{n}^{\circ} 3$ air-filled ) in the parallel polarization

551Figure 7: Analysis of the synthetic radargram (in pixels) associated with 3 pipes in the 552parallel configuration using the template-matching algorithm: (a) template defined, (b) $\mathrm{L}_{1}$ 553norm distance map calculated, and (c) positions of the image template detected in the original 554Bscan

555Figure 8: Analysis of the synthetic radargram (in pixels) associated with 3 pipes in the end556fire configuration using the template-matching algorithm: (a) template defined, (b) $\mathrm{L}_{1}$ norm 557distance map calculated, and (c) positions of the image template in the original Bscan

558Figure 9: Geometry of the sandy box with two buried objects, an air-filled pipe and a 559horizontal conductive strip, probed experimentally using a pair of bowtie slot antennas

560Figure 10: Synthetic template issued from FDTD simulations involving a pair of bowtie slot 561antennas (end-fire configuration) moved on a soil ( $\varepsilon_{s}^{\prime}=3.5$, $\sigma_{\mathrm{s}}=0.01$ S.m ${ }^{-1}$, $\mathrm{h}_{\mathrm{s}}=10 \mathrm{~mm}$ ) 562 with a buried conductive pipe ( $\mathrm{R}=32 \mathrm{~mm}$ )

563Figure 11: Analysis of experimental radargrams (in pixels) associated with a air-filled pipe 564and a horizontal conductive strip using the template matching algorithm in the parallel (a, b) 565and end-fire (c, d) configurations ; (a, c) distance maps, (b, d) positions of the image template 566in the original Bscans and hyperbola fitting

567Figure 12: (a) Mean distance map in the parallel and end-fire configurations associated with 568an air-filled pipe and a horizontal conductive strip, and (b) positions of the image template in 569the experimental Bscan of the end-fire configuration

570Figure 13: Analysis of experimental radargrams (in pixels) associated with (a) a water-filled 571pipe and a vertical conductive strip using the template matching algorithm in the end fire 572configuration ; (b) distance map, (c) positions of the image template in the original Bscan and 573hyperbola fitting

## 574LIST OF TABLES

575Table 1: Parameter estimation in the parallel configuration for the synthetic Bscan of Fig. 7c

576Table 2: Parameter estimation in the end-fire configuration for the synthetic Bscan of Fig. 8c 577Table 3: Parameter estimation in the parallel configuration for the experimental Bscan of Fig. 57811b

579Table 4: Parameter estimation in the end-fire configuration for the experimental Bscan of Fig. 58011d

581Table 5: Eigenvalues of the Hessian matrix at the estimates of the depth $d$ and velocity v from 582hyperbola fitting in both polarizations

