On Robust Fitting of Curves and Sets of Curves

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Abstract: Automatic road markings detection is a central point in road scene analysis from images. Its applications concern as well the inventory of lane marking types on a road network, as the design of driver assistance systems on-board vehicles. We here describe a model and an estimation algorithm which account for the geometric specificity and variability of the markings and, above all, which are robust to the numerous perturbations that can be observed in real-world images.

Usually, in a road image, several markings are to be detected, depending on the number of lanes. The two possible approaches consist in:

1. splitting the road image into parts and detect a single marking in each part,
2. fitting a complete road model that includes several road markings.

In both cases, anyway, the detection problem remains difficult. It may then be very profitable to introduce prior information in the form of both a road marking model and a statistical model of perturbations, or noise. Our approach involves a *local markings extractor* algorithm, whose outputs, namely putative road marking centers, are approximated by curves in a second step. The detection task is then set as the problem of recovering one or several instances of a curve model from a single noisy data set D that can be subject to severe perturbations, called *outliers*.

The road marking model must be as simple as possible, yet it must also put up with the variability of markings aspect in real-world scenes. We propose several functional bases that are suitable for this task. The model itself is linear with respect to its parameters, which contributes to the simplicity of the approach. This linear, deterministic *generative* model is associated with a noise model, b, that accounts for the perturbations of the data, or outliers. Each individual curve is thus explicitly described by a vector of parameters $\tilde{A}_j = (\tilde{a}_{i,j})_{0 \leq i \leq d}$, $1 \leq j \leq m$ following:

$$y = X(x)^t \tilde{A}_j + b \quad (1)$$

where $(x, y) \in \mathrm{D}$ denotes the image coordinates of a data point, $d+1$ is the number of curve parameters, m is the number of curves and $X(x)$ is the vector of basis functions at x (in the usual polynomial case $X(x) = (1, x, x^2, \ldots, x^d)^t$).

We assume that the measurement noise b is independent and identically distributed (i.i.d.). Since the Gaussian assumption is sensitive to outliers, a better noise model [1] is:

$$p_s(b) \propto \frac{1}{s} e^{\frac{-1}{2}\phi((\frac{b}{s})^2)} \tag{2}$$

where $\propto$ denotes the equality up to a factor and s is the scale of the probability density function (pdf). As stated by Huber [2], the role of ϕ is to saturate the error in case of a large noise and thus, to lower the importance of outliers.

Using a non-Gaussian noise model leads to a non-convex optimization problem. Indeed, the robust fitting of several curves is set as:

$$Min_A e(A) = \sum_{i=1}^{i=n} -\ln(\sum_{j=1}^{j=m} e^{-\frac{1}{2}\phi((\frac{X_i^t A_j - y_i}{s})^2)}) + n\ln(s) \tag{3}$$

where $(x_i, y_i) \in \mathrm{D}$, $1 \leq i \leq n$ are data points. Using the so-called Half Quadratic approach, and under certain assumptions on ϕ, we are able to reformulate (3) as a constrained minimization problem by introducing auxiliary variables w_{ij}:

$$\begin{cases} Min_A e(A) = \sum_{i=1}^{i=n} -\ln(\sum_{j=1}^{j=m} e^{-\frac{1}{2}\phi(w_{ij})}) + n\ln(s) \\ w_{ij} = (\frac{X_i^t A_j - y_i}{s})^2, 1 \leq i \leq n, 1 \leq i \leq m \end{cases} \tag{4}$$

In order to solve this problem, we use Lagrangian formalism, and a Difference of Convex functions (DC) or Convex-Concave problem is thus obtained. A deterministic, alternated minimization algorithm, for Simultaneous Multiple Robust Fitting (SMRF) algorithm is then derived. This algorithms may be applied for one or several curves and thus can be seen as an extension of the so-called Iterative Re-weighted Least Squares (IRLS) algorithm, widely used in M-estimators. Compared to IRLS, the SMRF algorithm features an extra probability ratio, which is classical in clustering algorithm, in the expression of the weights. Numerical issues are tackled by banning zero probabilities in the computation of the weights and by enforcing a Gaussian prior to the curve coefficients. Such a prior is, moreover, well-suited to sequential image processing and provides control on the curves as illustrated on processed sequences of real-world images.

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2. P. J. Huber. *Robust Statistics*. John Wiley and Sons, New York, New York, 1981.